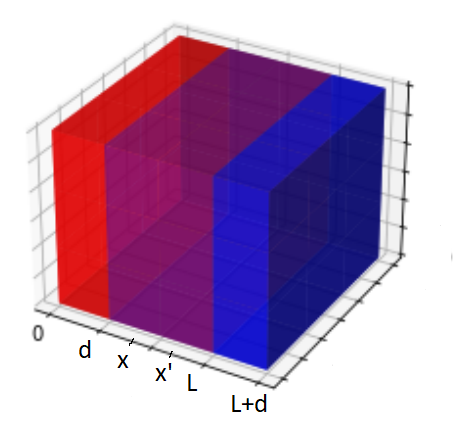
**Electric Susceptibility**

We can calculate a very basic formula, valid in the small q limit, for the electric susceptibility of our ionic lattice. It’s valid in the small q limit (technically just the q = 0 limit) because we’ll be considering the lattice as a rigid body, so that everything moves in unison. Basically, therefore, we will be calculating the susceptibility presuming that the only excitation the lattice can have is the plasma oscillation frequency ΩE, which we derived in the Excitations folder.

So calculating the sucsceptibility really requires quantum mechanics and statistical mechanics, but we can get away with classical mechanics here, and apropos stat mech, I’ll be assuming the ionic lattice is one state, the ground state I guess, and so there will be no need for thermodynamic averaging. I’m basically going to copy the work we did getting the susceptibility for the electrons in the metal, in the Electrodynamics/Metal model (TD). The only thing that changes is instead of considering the electrons to be moving, we have the positive lattice move. So we have a positively charged blue cube (playing the role of our metal’s positive ionic lattice) presently between (0,L) with negatively charged red cube (playing the role of our metal’s electrons) overlaid (purple is overlap) presently between (d,L+d). Each has equal/opposite charge density ρ, and equal opposite charge Q. Want to calculate the force the electron background substrate exerts on the positive ionic lattice embedded within it, when it is displaced by distance d (as it presently is, in the diagram below).



So the force, dF, a strip of the negative substrate at x and of width dx, exerts on the positive ionic lattice would be:



And then the force the entire positive substrate exerts on the entire negative electron gas overlaid would be:



We can see the the force is proportional to displacement. This means of course that if the ionic lattice uniformly displaces any tiny amount, it will oscillate back and forth about the electron gas substrate with a frequency given by ω = √(k/M), where k is the effective spring constant ρQ/ε0, and M is the mass of the total ionic lattice. So,



where n is the ion number density. And now we can write the force as:



Now let’s consider the motion of the ion block in an external EM field. According to Newton’s 2nd law, if we place our ion block in an external (free) electric field, we have:



We can solve this equation by taking the FT of both sides:



Now we can get the polarization density P(t). For small displacements, this is just:



So,



And from our study of insulators in the EM folder, we know that the polarization and electric field are related via:



So we can say,



We’ll recognize the prefactor as ΩE2. So we can say,



And from this, it follows that the dielectric function is:



So,



which is, I believe, the correct expression, at this level of approximation. For typical metals, the ionic plasma frequency is ΩE ~ 1013 Hz, which puts it in the infrared range. Note, in faux Gaussian units, this would read the same but with ΩE = √(4πρe/mion).

Finally, even though our model here predicts acoustic oscillations with a finite value in the small q (q = 2π/λ) limit, we’ll see that electrons will screen the interatomic potential that’s keeping the lattice in a sort of rigid body state, and thereby soften it up. This will have the effect of making the acoustic oscillation frequency go to 0 as q goes to 0. And just to be clear, by ‘acoustic oscillations’ we mean those for which every atom in a given cell basis moves in unison with every other atom in that basis. For instance if we were dealing with NaCl, then the two Na+ and Cl- atoms would move in unison with each other in an acoustic oscillation.

A picture containing timeline

Description automatically generated